However, if the power available does remain constant for speeds as low as V_{mp}, and one has $C_{L_{\mathrm{mp}}} < C_{L_e}$, then the maximum climb rate is given by

$$V_w = V_{\mathrm{mp}} = (\lambda_p \lambda_s / 3)^{1/4} / \sqrt{\sigma}, \qquad w = [\phi P_m - (\mathrm{mp})_0 / \sqrt{\sigma}] / W$$

$$(\mathrm{mp})_0 = (4W/3^{3/4})(\lambda_s^{3/4} \lambda_p^{-1/4}) = (2/3^{3/4}) D_* (V_*)_{\sigma=1} \tag{28}$$

where $(\mathrm{mp})_0$ is the minimum power required at sea level. The absolute ceiling is given by the numerical solution of $\phi(\sigma)$ and $\sigma(h)$ from the following equation for $w=0$

$$\phi_h P_m = (\mathrm{mp})_0 / \sqrt{\sigma_h} \tag{29}$$

For convenience, Anderson[6] assumed $\phi = \sigma$ in his numerical calculations, which of course are in exact agreement with Eq. (28) and now Eq. (29) reduces to a simple explicit relation for the value of σ at the absolute ceiling, namely,

$$\sigma_h = [(\mathrm{mp})_0 / P_m]^{2/3} \tag{30}$$

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Influence of Landing Gear Flexibility on Aircraft Performance During Ground Roll

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Nomenclature

a	= acceleration during ground roll
C_D	= aircraft drag coefficient, including ground effect
D	= total drag of the aircraft, including ground effect
g	= gravitational acceleration
I	= inertia force on the aircraft during ground roll
k_0, k_1, k_2, k_3	= landing gear deflection constants defined in Eqs. (4) and (5)
L_{HT}	= horizontal tail lift
L_{WB}	= aircraft lift without tail contribution
MAC	= mean aerodynamic chord
M_D	= moment about the main wheel contact point (mwcp) due to drag
M_{HT}	= moment about mwcp due to tail lift
M_I	= moment about mwcp due to inertia
M_L	= moment about mwcp due to aircraft lift
M_{R_N}	= moment about mwcp due to ground reaction at the nose gear
M_T	= moment about mwcp due to thrust
M_W	= moment about mwcp due to aircraft weight
M_0	= aerodynamic pitching moment about the aircraft aerodynamic center
R_M	= ground reaction on the main landing gear
R_N	= ground reaction on the nose landing gear
T	= thrust
W	= aircraft takeoff weight
Y_t, Y_s	= tire deflection and wheel axle travel defined in Eqs. (4) and (5)
μ	= coefficient of ground (runway) friction (~0.035)
Θ	= ground incidence

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Introduction

IT is generally assumed (see, for example, Ref. 1) that the attitude of the aircraft remains constant during ground roll and, consequently, C_L, C_D and other aerodynamic coefficients also remain constant during this phase. However, the flexibility of the landing gear system appreciably alters the attitude of the aircraft during ground roll. An analysis of the influence of landing gear deflection characteristics on the aircraft performance on the ground up to rotation is presented in this paper.

Problem Formulation

In order to rotate the aircraft about its main wheels, the horizontal tail has to overcome moments produced by the following forces about the main wheels: aircraft weight; thrust; inertia; ground reactions on the wheels; and the aerodynamic forces and moments, that is, lift, drag, and pitching moment of the aircraft. These are shown schematically in Fig. 1. A quasisteady dynamic equilibrium state has been assumed. The following simplifying assumptions have also been made: 1) calm air conditions; 2) C_{M_0} and C_D of the tail unit are negligible; 3) the aircraft lift and drag acting through the aerodynamic center are respectively normal and parallel to the horizontal ground plane. The ground incidence Θ is defined as the angle made by the mean aerodynamic chord of the wing with respect to the ground plane. The following equations for force and moment balance determine the quasiequilibrium conditions for the aircraft during its ground roll:

Forces normal to the ground:

$$L_{WB} + L_{HT} + (R_N + 2R_M) + T\sin\Theta = W \tag{1}$$

Forces parallel to the ground:

$$T\cos\Theta - D - \mu(R_N + 2R_M) = (W/g)a = I \tag{2}$$

Moment about the main wheel contact point with the ground:

$$M_W + M_0 + M_T + M_L + M_D + M_I + M_{R_N} = M_{HT} \tag{3}$$

In Eqs. (1-3) the unknowns are R_N, R_M, and L_{HT}. All others are known for a given aircraft attitude and speed. However, the aircraft attitude itself depends on the deflections of the landing gears which, in turn, are functions of R_N and R_M. It is possible to express the landing gear deflection in the vertical direction as a function of normal reaction as follows:

Tires:

$$Y_t = k_t R \tag{4}$$

Wheel axle travel:

$$Y_s = k_0 + k_1 R + k_2 R^2 \tag{5}$$

Table 1 Ground reactions and incidence during ground roll

R_N, kg	R_M, kg	Thrust, kg	Tail load $-L_{HT}$, kg	Speed, km/h	Ground incidence Θ, deg	Remarks
2,500	6,250	0	0	0	3.60	Aircraft static on the ground
4,500	8,500	6,000	0	0	0.75	Full thrust on, but aircraft braked
6,500	10,000	7,500	0	0	−1.0	Hypothetical case, with 25% "excess" thrust
2,800	3,600	6,000	−	300	2.3	Zero tail load at rotation speed
1,800	4,200	6,000	1,000	300	3.0	−
900	4,600	6,000	2,000	300	4.6	−
0	5,000	6,000	3,000	300	5.2	Nose wheel liftoff with 3000 kg down load at the tail[a]

[a] In order to develop this load, the C_{LT} of the tail was 1.23. If Θ is assumed constant at 3.6 deg, C_{LT} required = 1.53 for nose-wheel liftoff. In these computations the tail load has been assumed to be maintained zero right up to the rotation speed. Variations in tail load during ground roll will, of course, give rise to additional variations in R_N, R_M, and Θ.

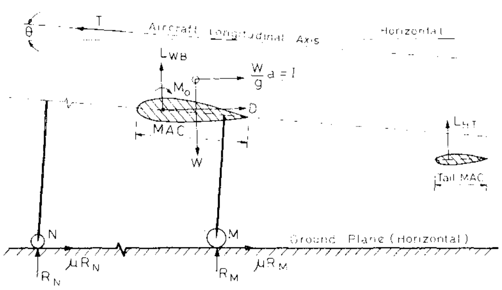

Fig. 1 Forces during ground roll.

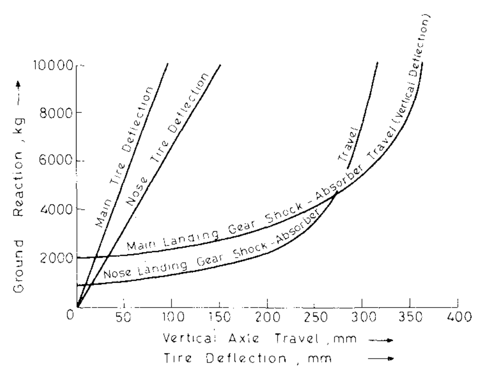

Fig. 2 Variation of landing gear deflection with ground reaction.

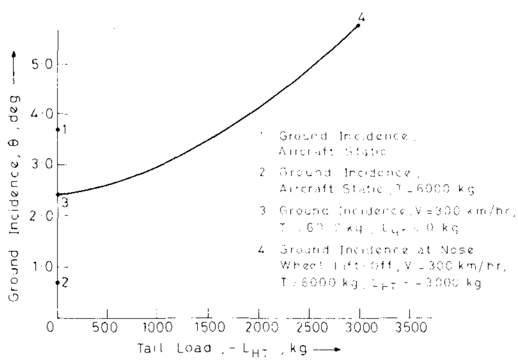

Fig. 3 Variation of aircraft attitude during ground roll.

where k_t is a constant that depends on inflated tire pressure; k_0, k_1, k_2 are known constants of the landing gear shock absorber system; and R is the normal ground reaction.

The actual values of the above constants are different for the nose and the main landing gears, the former being generally more flexible. A typical variation of deflections with normal reaction is shown in Fig. 2. The differential deflection of the nose and the main landing gears results in the variation of aircraft attitude which can be determined once the layout of the landing gear system is known.

Results and Discussion

Since the aerodynamic coefficients and the ground incidence Θ are mutually coupled through Eqs. (1-5), a numerical iterative solution is sought. Results of computation carried out for a typical 15 T class fighter aircraft have been presented in Table 1 and Fig. 3.

The aircraft attitude is initially positive since the c.g. is close to the main landing gear which has to bear most of the aircraft weight and hence deflects more. However, when the full thrust (6000 kg) is on, with the aircraft braked, the ground incidence Θ drops to 0.75 deg from the initial value of 3.6 deg. The value of Θ becomes even negative (−1.0 deg) when we consider a hypothetical case of the same powerplant developing a 25% excess thrust. If the aircraft is accelerated from such a condition (with Θ = −1.0 deg), with the tail producing an upload, as is the usual pilot technique during the initial phase of the ground roll, it is clear that the wing would develop negative lift leading to a further reduction in Θ. Such a situation could easily result in a nose-wheel damage or even a catastrophic accident.

The variation in aircraft attitude at various stages of ground roll is shown in Fig. 3. The tail load required for nose-wheel liftoff (rotation) was computed to be about 3000 kg and the corresponding value of Θ, 5.2 deg. The tail lift coefficient was 1.23. However, if the landing gear is assumed rigid, the corresponding value of C_{LT} was found to be 1.53, nearly 25% higher, indicating that the landing gear flexibility reduces the tail size requirements for aircraft rotation.

Conclusions

Results of numerical studies indicate that the landing gear deflections lead to an appreciable increase in the angle of attack during the ground roll and the load requirements on the horizontal tail are consequently much less to rotate the aircraft. The variation in the ground incidence due to landing gear flexibility could be as much as ±50% and the reduction in tail load requirements almost 25%.

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